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INTERNAL FUNDAMENTAL SEQUENCES AND APPROXIMATIVE RETRACTS

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We introduce the notion of internal fundamental sequence and prove that any shape morphism from an arbitrary compactum X to an internally movable compactum Y is induced by an internal fundamental sequence. We use this special kind of fundamental sequences to give characterizations and some properties of AANR_C-sets and AANR_N-sets. The paper ends with a section devoted to internal FANR's.

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internal movability	AANR_N	internal FANR
internal fundamental sequences	FANR	U-subsequence.

0. Introduction

Since H. Noguchi [18] introduced in 1953 a generalization of absolute neighborhood retracts, many other topologists have contributed to develop a theory of approximative retraction. Some properties of AANR_N-sets (Approximative Absolute Neighborhood Retracts in the sense of Noguchi) were studied by A. Gmurczyk [12] and A. Granas [13]. In 1971, M.H. Clapp [8] generalized Noguchi's notion and introduced a larger class of spaces which are known under the name of AANR_C-sets (Approximative Absolute Neighborhood Retracts in the sense of Clapp). AANR_C-sets and related concepts were studied among others by S.A. Bogatyĭ [1], K. Borsuk [3], L. Boxer [4], J. Dydak–J. Segal [11] S. Mardešić [16], P.R. Patten [19], etc. Recently, Z. Čerin [5], [6], [7] and T. Watanabe [21] obtained a number of properties of AANR_C-sets and AANR_N-sets, by using methods that are inspired by Borsuk's shape theory. The notion of internal movability was introduced by Bogatyĭ [1], who proved that AANR_C-sets are internally movable. J. Dydak proved [9] that each movable compactum has the shape of an internally movable compactum.

In this paper we see that any shape morphism from an arbitrary compactum X to an internally movable compactum Y is induced by an internal fundamental sequence. Hence, if we limit ourselves to the class of internally movable compacta, shape theory may be developed in terms of internal fundamental sequences. We use this special kind of fundamental sequences to give characterizations and some properties of $AANR_C$-sets and $AANR_N$-sets. The paper ends with a section devoted to internal FANR's. We assume that the reader is familiar with some elementary notions from the theory of approximative retraction and shape theory (see [1], [8], [2], [10], [17]).

1. Internal fundamental sequences

Definition 1.1. Let X and Y be compacta lying in spaces $M, N \in AR$, respectively. A fundamental sequence $f = \{f_k, X, Y\}_{M,N}$ is said to be internal provided $f_k(X) \subset Y$, for each $k = 1, 2, \dots$

Remark 1.1. In [6] Z. Čerin defines the notion of a net from X into Y (in M, N) as a sequence $f_k : (M, X) \to (N, Y)$ of maps of pairs satisfying the following condition:

For every neighborhood V of Y in N, there exists a neighborhood U of X in M such that $f_k(U) \subset V$ for almost all k.

So, an internal fundamental sequence is the same as a fundamental sequence which is a net.

Proposition 1.1. *If the compactum Y is internally movable, then every fundamental sequence $f = \{f_k, X, Y\}_{M,N}$ is homotopic to an internal fundamental sequence.*

Proof. Let $V_1 \supset V_2 \supset \cdots \supset V_n \supset V_{n+1} \supset \cdots$ be a basis of neighborhoods of Y in N. Due to the internal movability of Y, it follows that, for every index n, there exists a neighborhood $W_n \subset V_n$, of Y in N, and there exists a map $s_n : W_n \to Y$, such that:

$$s_n \simeq j_n \quad \text{in } V_n, \tag{1}$$

where j_n is the inclusion of W_n into V_n. Since f is a fundamental sequence, we can find a sequence of indexes $m_1 < m_2 < \cdots < m_n < m_{n+1} < \cdots$ such that $f_k|_X \simeq f_{k+1}|_X$ in $W_n \subset V_n$, for every $k \geqslant m_n$.

Now we define a sequence of maps $g'_k : X \to Y$ as follows: $g'_1, \dots, g'_{m_1 - 1}$ are arbitrary (continuous) maps; $g'_k = s_n \cdot f_k|_X$, provided $m_n \leqslant k < m_{n+1}$. For every neighborhood V of Y in N, there exists an index n_0 such that $V_{n_0} \subset V$, and if $k \geqslant m_{n_0}$, there exists another index $n \geqslant n_0$ such that $m_n \leqslant k < m_{n+1}$ hence $g'_k = s_n \cdot f_k|_X$, and from (1) it follows that:

$$g'_k \simeq f_k|_X \quad \text{in } V_n \subset V_{n_0} \subset V. \tag{2}$$

Since $f_k|_X \simeq f_{k+1}|_X$ in V_{n_0}, we conclude that $g'_k \simeq g'_{k+1}$ in $V_{n_0} \subset V$ for $k \geq m_{n_0}$. Now, following Mardešić [15, p. 1133], there exists a fundamental sequence $g = \{g_k, X, Y\}_{M,N}$ such that $g_k|_X = g'_k$. Then, g is an internal fundamental sequence and (2) implies that $g \simeq f$. This completes the proof.

As a consequence of Proposition 1.1, in the class of internally movable compacta, the notion of shape can be defined in terms of internal fundamental sequences.

Remark 1.2. It is not in general true that every internal fundamental sequence is homotopic to a fundamental sequence generated by a map (even in the case of internally movable compacta), as the following example shows: Consider in the Euclidean plane E^2 the circles $X_n = \{(x, y) \mid x^2 + y^2 = (1 - 1/n+1)^2\}$ for $n \geq 1$ and let X_0 be the circle with centre $(0, 0)$ and radius 1. We set $X = \bigcup_{n=0}^{\infty} X_n$. Now, consider the points $p_1 = (0, 0)$, $p_{2m} = (1 - 1/m+1, 0)$, $p_{2m+1} = ((1/m+1) - 1, 0)$ for each $m \geq 1$, and let $(Y_n)_{n \geq 0}$ be a sequence of mutually disjoint circles such that $Y_0 = X_0$, the centre of Y_n is p_n (for $n \geq 1$) and lim, radius$(Y_n) = 0$. We set $Y = \bigcup_{n=0}^{\infty} Y_n$. It is easy to see that X and Y are internally movable compacta.

Consider a fundamental sequence $f = \{f_k, X, Y\}_{E^2, E^2}$ such that, for every $k \geq 1$, f_k maps homeomorphically X_i onto Y_i, if $1 \leq i \leq k$, and maps X_0 and X_i to the point $p = (1, 0)$, if $i > k$. f is an internal fundamental sequence and we now prove that the fundamental class of f is not generated by a map.

Otherwise, if f were homotopic to a fundamental sequence, g, generated by a map $g: X \to Y$, then, the induced maps, Λ_f, Λ_g, between the spaces of components $\square(X)$ and $\square(Y)$ would be coincident (see Borsuk [2], p. 214), and we should have $\Lambda_g(X_n) = \Lambda_f(X_n) = Y_n$, and, therefore $g(X_n) \subset Y_n$ for every $n \geq 0$. Let $x_n \in X_n$ be the point $(1 - 1/n+1, 0)$ for each index $n \geq 1$; then, the sequences (x_{2n}) and (x_{2n+1}) converge to the point p but lim. $g(x_{2n}) = p$, lim. $g(x_{2n+1}) = p' = (-1, 0)$. This contradicts the fact that g is continuous.

A theory of shape for compacta, parallel to that of Borsuk can be defined by means of internal fundamental sequences. We give here the basic definition and omit all details.

Definition 1.2. A compactum X lying in $M \in AR$ is said to be internally shape equivalent to a compactum Y which lies in $N \in AR$ provided there exist internal fundamental sequences $f = \{f_k, X, Y\}_{M,N}$ and $g = \{g_k, Y, X\}_{N,M}$ such that $g \cdot f \simeq i_{X,M}$ and $f \cdot g \simeq i_{Y,N}$. If we assume only that the relation $g \cdot f \simeq i_{X,M}$ holds, then we say that X is internally shape dominated by Y.

The next result generalizes Bogatyĭ's Theorem 3, [1]. We omit the proof because it can be obtained by a small modification on that of Borsuk's Theorem [2, p. 150], which establishes that movability is a hereditary shape property.

Proposition 1.2. Let X and Y be compacta lying in spaces $M, N \in AR$ respectively and suppose that X is internally shape dominated by Y. If Y is internally movable, then X is also internally movable.

2. Approximate ANR's

In this Section we state some properties of AANR's in terms of internal fundamental sequences. All spaces mentioned here are compacta and, for the sake of simplicity they are assumed to lie in the Hilbert cube, Q. By the symbol d we mean the usual distance in Q.

Definition 2.1. Two fundamental sequences $f = \{f_k, X, Y\}$ and $\mathbf{g} = \{g_k, X, Y\}$ are said to be ε-close provided $d(f_k(x), g_k(x)) < \varepsilon$, for every point $x \in X$, and for almost all k; if the above inequality holds for every index k, we say that f and $\mathbf{g}$ are strictly ε-close.

Proposition 2.1. *Let Y be an* AANR_C*-set. Then, for every fundamental sequence $f = \{f_k, X, Y\}$ there exists an internal fundamental sequence $\mathbf{g} = \{g_k, X, Y\}$ such that f and $\mathbf{g}$ are ε-close for every $\varepsilon > 0$.*

Proof. Consider a sequence $V_1 \supset V_2 \supset \cdots \supset V_n \supset V_{n+1} \supset \cdots$ of ANR neighborhoods of Y in Q, shrinking to Y. Then, there exists a null sequence of positive numbers $\varepsilon_1 > \varepsilon_2 > \cdots > \varepsilon_n > \varepsilon_{n+1} > \cdots$, such that any two ε_n-close maps from an arbitrary space into V_n are homotopic. Since Y is an AANR_C-set, there exist, for every index n, a neighborhood U_n of Y in Q, and a map $s_n : U_n \to Y$, such that $d(s_n(y), y) < \varepsilon_n$, for every $y \in U_n$. Besides we may assume that $U_{n+1} \subset U_n \subset V_n$ for each index n. Since f is a fundamental sequence, there exists a sequence of indexes $k_1 < k_2 < \cdots < k_n < k_{n+1} < \cdots$ such that:

$$f_k|_X \simeq f_{k+1}|_X \quad \text{in } U_n, \text{ for every } k \geq k_n. \tag{1}$$

Now, consider the sequence of maps $g'_k : X \to Y$ defined by: $g'_1, \ldots, g'_{k_1-1}$ are arbitrary (continuous) maps; $g'_k = s_n \cdot f_k|_X$ provided $k_n \leq k < k_{n+1}$. Let n be an arbitrary index, and let $k \geq k_n$, and suppose $k_m \leq k < k_{m+1}$, where m is an index, $m \geq n$. Since $d(g'_k(x), f_k(x)) = d(s_m(f_k(x)), f_k(x)) < \varepsilon_m \leq \varepsilon_n$, for every $x \in X$, we have that:

$$g'_k \simeq f_k|_X \quad \text{in } V_n, \text{ for every } k \geq k_n. \tag{2}$$

For every neighborhood V of Y in Q, there exists an index n_0 such that $V_{n_0} \subset V$. Then, applying (1) and (2), we deduce that $g'_k \simeq g'_{k+1}$ in V, for every $k \geq k_{n_0}$. By the previously mentioned Mardešić's result [15], there is a fundamental sequence $\mathbf{g} = \{g_k, X, Y\}$ such that $g_k|_X = g'_k$ for every index k. Obviously, $\mathbf{g}$ is internal and for every $\varepsilon > 0$, there exists an index n_1 such that $\varepsilon_{n_1} < \varepsilon$; therefore, for every

$k \geq k_{n_1}$, and for every point $x \in X$, the following inequalities hold: $d(g_k(x), f_k(x)) < \varepsilon_{n_1} < \varepsilon$, hence f and g are ε-close, and the proof is finished.

The next result is a characterization of $\mathrm{AANR_N}$-sets in terms of internal fundamental sequences.

Proposition 2.2. *A compactum X is an $\mathrm{AANR_N}$-set if and only if there exist a closed neighborhood U of X in Q and an internal fundamental sequence $f = \{f_k, U, X\}$ such that, the restricted fundamental sequence $f|_X = \{f_k, X, X\}$ is ε-close to the identity $\boldsymbol{i}_X$ for every $\varepsilon > 0$.*

Proof. First, suppose that there exists a fundamental sequence $f: U \to X$ with the required properties and let $\varepsilon > 0$ be given. Then, there exists an index k such that $d(f_k(x), x) < \varepsilon$ for every $x \in X$. Hence, $f_k|_U : U \to X$ is an ε-retraction and X is an approximative retract of U. Therefore, X is an $\mathrm{AANR_N}$-set.

Let us prove now the part 'if' of the proposition. By Gmurczyk's Corollary [12, p. 63] (see also Bogatyĭ [1]), X is an FANR-set. So, there exists a fundamental retraction $\boldsymbol{r} = \{r_k, U, X\}$, from a closed neighborhood U of X in Q, to X. By Proposition 2.1, there exists an internal fundamental sequence $f = \{f_k, U, X\}$. such that $\boldsymbol{r}$ and f are ε-close, for every $\varepsilon > 0$. Then, if $\varepsilon > 0$ is given, we have that $d(f_k(x), x) = d(f_k(x), r_k(x)) < \varepsilon$, for every $x \in X$ and for almost all k. Hence, f satisfies the required properties. This completes the proof.

Definition 2.2. A subsequence $\{f_{k_i}, X, Y\}$ of a fundamental sequence $f = \{f_k, X, Y\}$ is said to be an U-subsequence (where U is a neighborhood of Y in Q), provided $f_{k_i}|_X \simeq f_{k_{i+1}}|_X$ in U, for every $i \geq 1$.

Proposition 2.3. *For a compactum Y, the following conditions are equivalent:*

a) *Y is an $\mathrm{AANR_C}$-set.*

b) *Y is movable and for every $\varepsilon > 0$, there exists a neighborhood $U = U(\varepsilon)$ of Y in Q, such that, for every fundamental sequence $f = \{f_k, X, Y\}$, every U-subsequence of f is strictly ε-close to an internal fundamental sequence.*

Proof. a)$\Rightarrow$b) It is well known that if Y is an $\mathrm{AANR_C}$-set, then Y is movable (indeed, it is internally movable: see Bogatyĭ [1]). Moreover, for every $\varepsilon > 0$, there exists a map $r: U \to Y$, from a closed neighborhood U of Y in Q, to Y, such that $d(r(y), y) < \varepsilon$, for every $y \in U$. Consider a fundamental sequence $\boldsymbol{r} = \{r_k, U, Y\}$ generated by r, and a fundamental sequence $\boldsymbol{j} = \{j_k = i_Q, Y, U\}$ generated by the inclusion j of Y into U. Then, for every fundamental sequence $f = \{f_k, X, Y\}$ and every U-subsequence $f' = \{f_{k_i}, X, Y\}$, the composition $\boldsymbol{r} \cdot \boldsymbol{j} \cdot f' = \{r_i \cdot j_i \cdot f_{k_i}, X, Y\}$ is an internal fundamental sequence strictly ε-close to f'.

b)$\Rightarrow$a) Let $\varepsilon > 0$ be given, and consider a neighborhood U of Y in Q such that every U-subsequence of each fundamental sequence from an arbitrary compactum to Y, is strictly ε-close to an internal fundamental sequence. Take another (closed) neighborhood V of Y in Q, contained in the interior of U. Since Y is uniformly movable (see Spiez [20]), there exists a fundamental sequence $\boldsymbol{f}': U_0 \to Y$, from a closed neighborhood $U_0 \subset V$ of Y in Q, to Y, such that $\boldsymbol{j} \cdot \boldsymbol{f}' \simeq \boldsymbol{i}$, where $\boldsymbol{j}: Y \to V$ and $\boldsymbol{i}: U_0 \to V$ are induced by the corresponding inclusions. It follows that, for the neighborhood U of V in Q, there exists an index k_0 such that $f'_k|_{U_0} \simeq i|_{U_0}$ in U for every $k \geqslant k_0$. Then, setting $f_1 = i_Q$ and $f_n = f'_{k_0+n-2}$, for every $n \geqslant 2$, we have a fundamental sequence $\boldsymbol{f} = \{f_k, U_0, Y\}$ which is an U-subsequence of itself, and the assumption b) implies that there exists an internal fundamental sequence $\boldsymbol{g} = \{g_k, U_0, Y\}$ strictly ε-close to f. Therefore $g_1|_{U_0}: U_0 \to Y$ is a map such that $d(g_1(y), y) = d(g_1(y), f_1(y)) < \varepsilon$ for every $y \in U_0$. Hence, $g_1|_{U_0}$ is an ε-retraction and Y is an $\mathrm{AANR_C}$-set. This completes the proof.

Definition 2.3. Two fundamental sequences $\boldsymbol{f} = \{f_k, X, Y\}$ and $\boldsymbol{g} = \{g_k, X, Y\}$ are said to be δ-homotopic (where δ is a given positive number) provided that for every neighborhood U of Y in Q, there exists an index k_0 such that for every $k \geqslant k_0$ there is a δ-homotopy $\psi_k: X \times [0, 1] \to U$, such that $\psi_k(x, 0) = f_k(x)$, $\psi_k(x, 1) = g_k(x)$, for every $x \in X$.

Remark 2.1. In [6]. Čerin gives a slightly different definition of δ-homotopic fundamental sequences. A δ-homotopy between $f_k|_{U'}$ and $g_k|_{U'}$ is required there, for every $k \geqslant k_0$, where U' is a neighborhood of X in Q, assigned to the neighborhood U of Y in Q.

Proposition 2.4. *Let Y be an* $\mathrm{AANR_C}$*-set and δ a positive number. Then, there exists an $\varepsilon > 0$, such that, for any two ε-close fundamental sequences $\boldsymbol{f}, \boldsymbol{g}: X \to Y$ defined on an arbitrary compactum X, there are internal fundamental sequences $\boldsymbol{f}', \boldsymbol{g}': X \to Y$, such that*:

a) *$\boldsymbol{f}'$ is δ-close to $\boldsymbol{f}$ and $\boldsymbol{g}'$ is δ-close to $\boldsymbol{g}$, and*

b) *$\boldsymbol{f}'$ and $\boldsymbol{g}'$ are δ-homotopic.*

Proof. Since Y is an $\mathrm{AANR_C}$-set, there exist a closed ANR neighborhood U of Y in Q, and a map $r: U \to Y$, such that $d(r(y), y) < \delta/3$ for every $y \in U$. Take an $\varepsilon > 0$ such that any two ε-close maps from an arbitrary compactum to U are $\delta/3$-homotopic (see Hu [14], p. 111). Consider now two ε-close fundamental sequences $\boldsymbol{f}, \boldsymbol{g}: X \to Y$. Then, there exists an index k_0, such that $f_k|_X$ and $g_k|_X$ are $\delta/3$-homotopic in U, for every $k \geqslant k_0$.

We define $f'_k = \hat{r} \cdot f_k$ and $g'_k = \hat{r} \cdot g_k$, for every $k \geqslant k_0$, where $\hat{r}: Q \to Q$ is a continuous extension of r and we take arbitrary continuous maps $f'_k, g'_k: Q \to Y$, if $k < k_0$. It is clear that $\boldsymbol{f}' = \{f'_k, X, Y\}$ and $\boldsymbol{g}' = \{g'_k, X, Y\}$ are internal fundamental sequences.

Moreover:

$$d(f'_k(x), f_k(x)) = d(r(f_k(x)), f_k(x)) < \delta/3$$

and

$$d(g'_k(x), g_k(x)) = d(r(g_k(x)), g_k(x)) < \delta/3$$

for every $x \in X$ and $k \geq k_0$.

Besides, from the fact that $f_k|_X$ and $g_k|_X$ are $\delta/3$-homotopic in U, we easily get that $f'_k|_X = r \cdot f_k|_X$ and $g'_k|_X = r \cdot g_k|_X$ are δ-homotopic for $k \geq k_0$. Indeed, for every $k \geq k_0$, let $\psi_k : X \times [0, 1] \to U$ be a $\delta/3$-homotopy between $f_k|_X$ and $g_k|_X$. Then, $r \cdot \psi_k : X \times [0, 1] \to Y$ is a homotopy between $f'_k|_X$ and $g'_k|_X$, such that, for every $x \in X$ and for every $t, t' \in [0, 1]$:

$$\begin{aligned} d(r \cdot \psi_k(x, t), r \cdot \psi_k(x, t')) &\leq d(r \cdot \psi_k(x, t), \psi_k(x, t)) \\ &\quad + d(\psi_k(x, t), \psi_k(x, t')) \\ &\quad + d(\psi_k(x, t'), r \cdot \psi_k(x, t')) \\ &< \frac{\delta}{3} + \frac{\delta}{3} + \frac{\delta}{3} = \delta. \end{aligned}$$

This completes the proof.

3. Internal FANR's

In this section we introduce the notion of an internal FANR.

Definition 3.1. A compactum X is said to be an internal FANR provided that for every compactum X' lying in an AR-set, M, and containing X, there exists an internal fundamental sequence $f = \{f_k, U, X\}_{M,M}$, from a (closed) neighborhood U of X in X', to X, such that:

$$f \cdot j \simeq i, \tag{1}$$

where $j: X \to U$ is induced by the inclusion and $i: X \to X$ is the identity fundamental sequence.

Remark 3.1. If, in the preceding definition, we do not require the fundamental sequence f to be internal, we merely have a necessary and sufficient condition for a compactum X to be an FANR-set.

Remark 3.2. No generality is lost if we restrict ourselves to the case $M = X' = Q$ (the Hilbert cube) in Definition 3.1.

Proposition 3.1. *A compactum X is an internal* FANR *if and only if it is an internally movable* FANR*-set.*

Proof. We may assume that X lies in Q. If X is an internal FANR, then, X is an

FANR-set (see Remark' 3.1). Moreover, there exists an internal fundamental sequence $f: U \to X$ from a closed neighborhood U of X in Q, to X, such that $f \cdot j \simeq i$, where $j = \{j_k = i_Q, X, U\}$ and $i: X \to X$ are like in (1).

It follows that, for every neighborhood $\hat{U}$ of X in Q, there exists a neighborhood $U_0 \subset U$ of X in Q, and an index k_0, such that $f_k \cdot j_k|_{U_0} = f_k|_{U_0} \simeq i|_{U_0}$ in $\hat{U}$, for every $k \geq k_0$. Hence, there exists a homotopy $\phi: U_0 \times [0, 1] \to \hat{U}$ such that $\phi(x, 0) = x$ and $\phi(x, 1) = f_{k_0}(x) \in X$, for every $x \in U_0$.

Then, X is internally movable and the part 'only if' is proved. Conversely, if X is an internally movable FANR-set, there exists a fundamental retraction $r = \{r_k, U, X\}$ where U is a closed neighborhood of X in Q. Then, Proposition 1.1 implies that r is homotopic to an internal fundamental sequence $f: U \to X$. Hence, $f \cdot j \simeq r \cdot j \simeq i$, and, as a consequence, X is an internal FANR and the proof is finished.

Proposition 3.2. *If a compactum X is internally shape dominated by an internal* FANR, *Y, then, X is an internal* FANR.

Proof. It is obvious from Propositions 1.2 and 3.1, and Borsuk's Theorem 2.1 [2, p. 255].

The next result is implicitly contained in [9].

Proposition 3.3. (Dydak). *Every fundamental class $[f]: X \to Y$, where Y is an internal* FANR, *is generated by a map.*

Proof. See [9], Corollary 3.

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